CENTRE
DE RECHERCHES
MATHÉMATIQUES

Méthodes topologiques en analyse non linéaire:développements récents -
Conférence à la mémoire du Professeur Andrzej Granas
4 - 8 juillet 2022

Topological Methods in Nonlinear Analysis: Recent Advances - Conference
in memory of Professor Andrzej Granas
July 4 - 8, 2022

Andrzej Szulkin
(Stockholm University)

Normalized solutions to non-variational Schr\"odinger systems

---

The elliptic system of 2 equations

$$\begin{cases} -\Delta u_i + \kappa_i u_i = \mu_i u_i^3 + 2\lambda u_i u_j^2, \quad i,j = 1,2,\ i \neq j, \\ u_i \in H^1(\mathbb{R}^N),\ |u_i|_2 = r_i > 0, \quad i = 1,2 \end{cases}$$

has been recently studied in dimensions $N \leq 4$. Here r_i are prescribed and κ_i are free (they appear as Lagrange multipliers). Such systems arise e.g. when studying mixtures of Bose-Einstein condensates or propagation of wave packets in nonlinear optics. The L^2-norms respectively represent the number of particles and power supply. Also various extensions (nonlinearities other than cubic, ℓ instead of 2 equations) have been recently studied. In all papers we know of the system is variational.

In this talk we will be concerned with a system of ℓ equations which in general is non-variational. If $\ell = 2$, the system we consider is

$$\begin{cases} -\Delta u_i + \kappa_i u_i = \mu_i u_i^p + \lambda_{ij} u_i^{\alpha_{ij}} u_j^{\beta_{ij}}, \quad i,j = 1,2,\ i \neq j, \\ u_i > 0,\ u_i \in H^1(\mathbb{R}^N),\ |u_i|_2 = r_i > 0, \quad i = 1,2, \end{cases}$$

where $2 \leq N \leq 4$, $\mu_i > 0$, $\lambda_{ij} > 0$ for all i,j or $\lambda_{ij} < 0$ for all i,j and $p, \alpha_{i,j}, \beta_{i,j}$ satisfy suitable assumptions. We show, using a combination of variational and topological arguments, that this system possesses at least one solution.

This is joint work with Monica Clapp.